Glassy dynamics in the hard matrix model

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The Hard Matrix Model

Introduction

For an orthogonal matrix $U \in \mathrm{SO}(N)$, Hamiltonian is proportional to its 1-norm:

$$H(U) = -\sqrt{N} \sum_{ij} |U_{ij}|$$

- No quenched disorder
- Naturally defined classical and quantum dynamics
- Easily treated with Monte Carlo
- Analytically tractable high- and low-temperature expansions

Has a low-temperature broken-symmetry phase obscured by ergodicity breaking.

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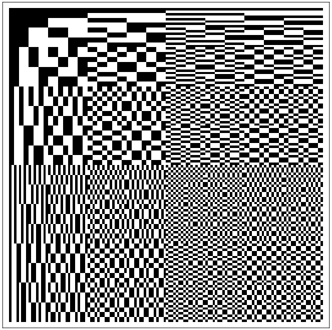
Ground states



For $N = 4m$, ground states are proportional to *Hadamard matrices* that have all elements ± 1

Combinatorially nontrivial: count only known up to $N = 32$, empirically grows as $\sim N^{1.6}$

Combinatorially hard to to produce: no example exists for $N = 668$

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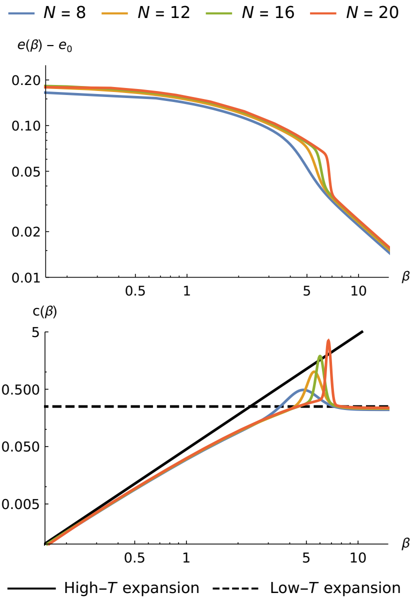
Equilibrium thermodynamics

Equilibrium model has abrupt phase transition at $\beta \simeq 7.9$

Landscape resembles harmonic well around minima

Low-temperature limit of entropy directly related to number of ground states

Monte Carlo accurately counts the $\sim 2 \times 10^{45}$ Hadamards for $N = 20$



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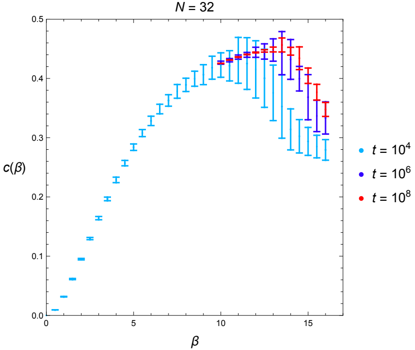
Metastable phenomena



For $N \gtrsim 28$, the ground state is not found in reasonable computer time and an abrupt transition does not occur

Relaxation-time dependent peak appears in the specific heat well above equilibrium transition temperature

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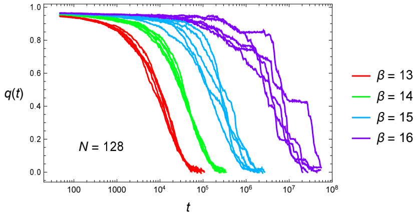
Metastable phenomena

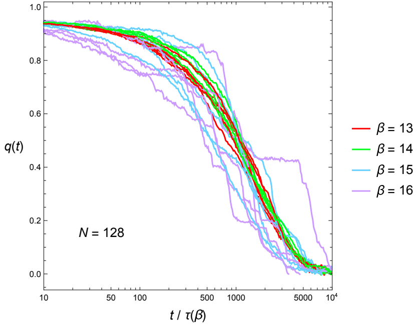
Super-exponential growth of mixing time with inverse temperature:

$$\tau(\beta) \propto \exp[\exp(\beta e_0 - b)]$$

Numerous critical points characterized by

$$U^T \operatorname{sign}(U) = \operatorname{sign}(U)^T U$$





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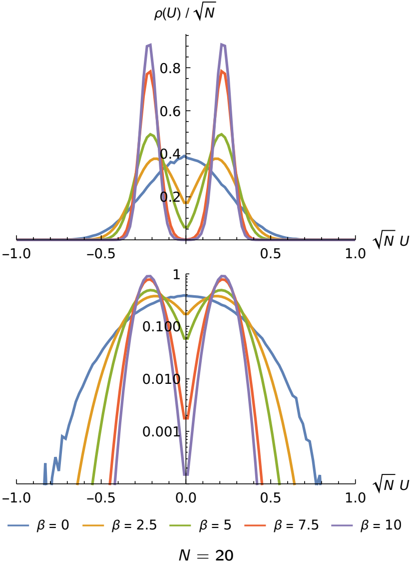
Transition state density

New conceptual tool: *the transition state density* ρ_0

Places where $U_{ij} = 0$ are sharp local maxima, passage through them connects states

The distribution $\rho(U)$ of matrix elements sees sharp feature around $\rho(0) \propto \rho_0$



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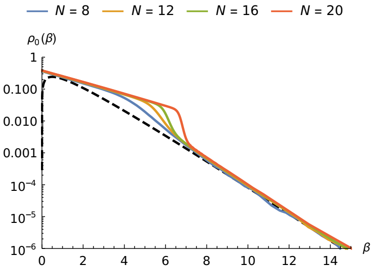
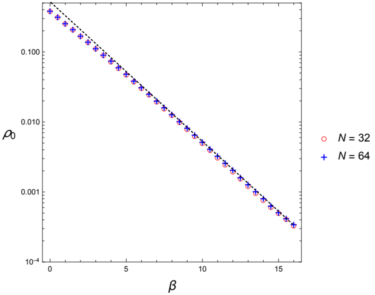
Transition state density

Transition state density jumps abruptly through transition to 'crystal'

For 'glass,' ρ_0 continues Arrhenius behavior of 'liquid'

Presence at arbitrarily low temperatures indicates the absence of a gap





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Conclusion

Simply posed, disorder-free, theoretically tractable model with arrested dynamics.

Many open questions:

- How many critical points are there, at what energies?
- Is a dynamical transition present?
- Does it 'age' after a quench, or show other 'glassy' physics?

Initial investigation: `arXiv:1912.07558 [cond-mat.stat-mech]`

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