Exploring the quasibrittle process zone with real-space RG

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Introduction

Understanding material cracking and fracture is necessary for understanding the aging and failure of those materials in our buildings and infrastructure. In ordinary brittle materials like glass, stress at the tip of a crack causes it to quickly and cleanly propagate through the material. In ductile materials like metals, this stress is reduced by plastic deformation around the crack tip, forming the crack's **process zone**. In quasi- (or disordered) brittle materials like concrete, this stress is reduced by opening a complicated network of microcracks in the process zone. This makes the structure of the quasibrittle process zone and crack propagation difficult to study by ordinary means.

Figure 1: Cracking in concrete.

Fuse Networks

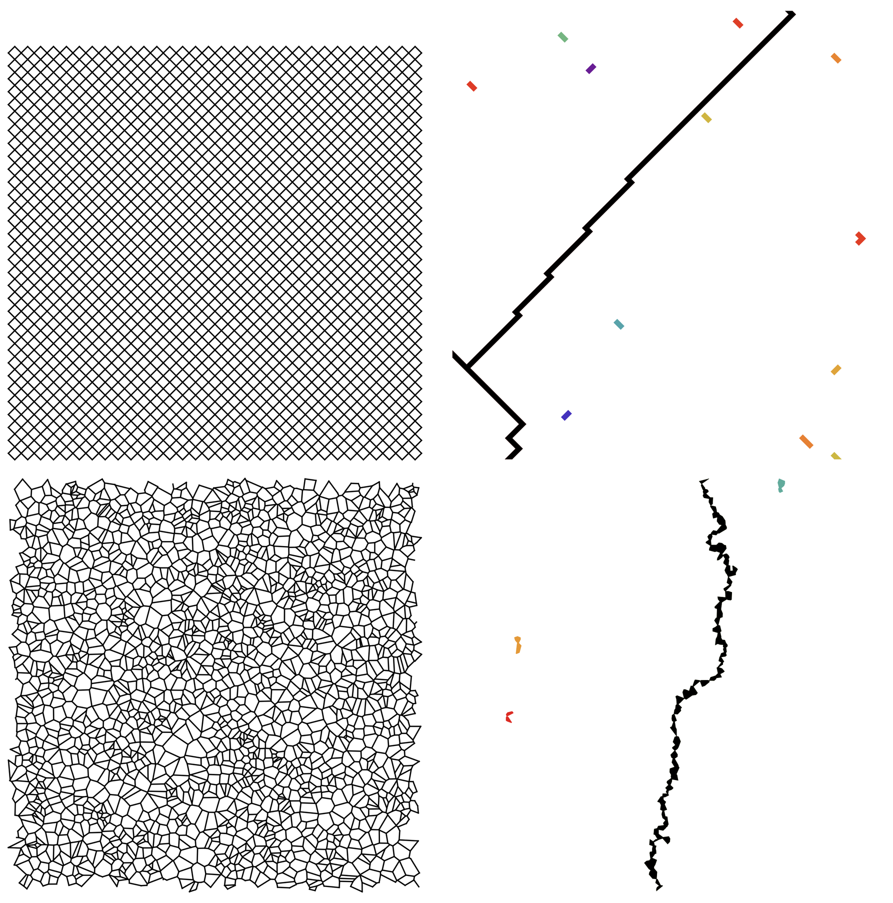

Figure 2: Contrasting the square (top) and voronoi (bottom) networks. **Left:** Unbroken fuse networks. **Right:** A fracture surface in each at low disorder ($\beta = 10$).

We model quasibrittle cracking and fracture using simulations of fuse networks, which are electrical systems of resistive fuses whose random current thresholds t are cumulatively distributed by t^β. β parameterizes the amount of disorder in the system: large β corresponds to vanishing disorder, small β to very large disorder. Fracture is performed adiabatically: the fuse whose ratio of current to threshold is largest breaks, and the current across the networks is recomputed. In order to reduce lattice effects, which become large for small disorder (see Figure 2), we use voronoi meshes for our fuse networks.

Homogeneous Scaling

The problem of fracture in fuse networks was unresolved until recently. For low disorder, fracture is nucleation-like, similar to that of ordinary brittle systems. At large disorder, fracture occurs after a very large amount of uncorrelated damage, and appears percolation-like. Sethna and Shekhawat developed a theory which unifies these behaviors with an RG crossover at intermediate disorder characterized by mean-field avalanches. The percolation-like behavior at high disorder was shown to be unstable under course-graining, and therefore any nonzero β will cause nucleated fracture at a sufficiently large system size (see Figure 3).

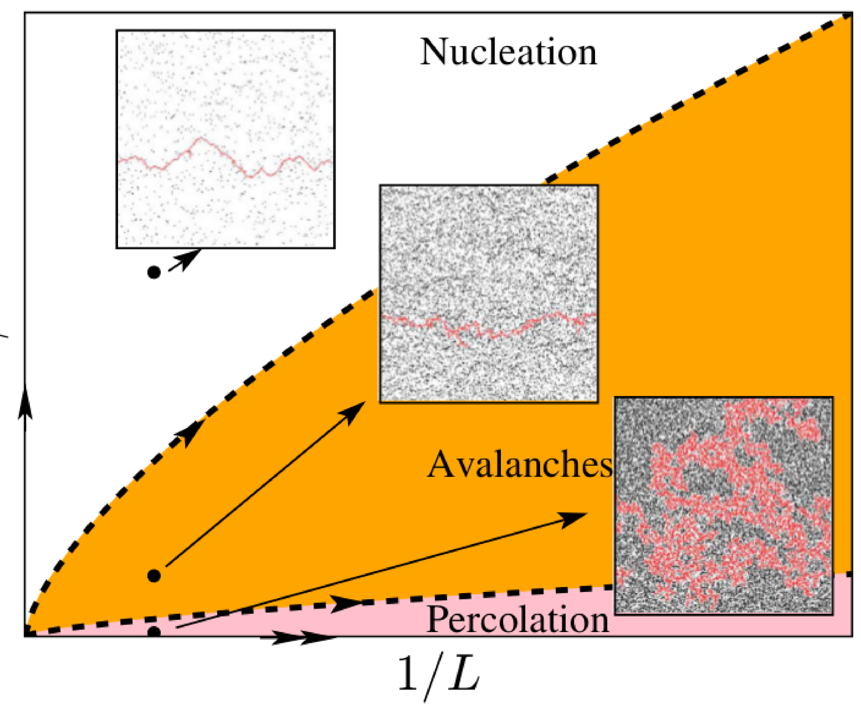

Figure 3: The 'phase diagram' for fracture in homogeneous systems.

In unpublished work, Shekhawat and Sethna found a scaling form for the distribution of network strengths $\sigma_{\max}$, the largest current applied to the network before it has broken. It is given by

$$P(\sigma_{\max} \mid \beta, L, u) = \sigma_{\max}^{-\tau_\sigma}\mathcal{P}(\beta L^{1/\nu_f}, \sigma_{\max}L^{\delta}, uL^{-\Delta/\nu_f}) \qquad (1)$$

where τ_σ, ν_f, δ and Δ are universal exponents, L is the system size, and u is an irrelevant scaling variable.

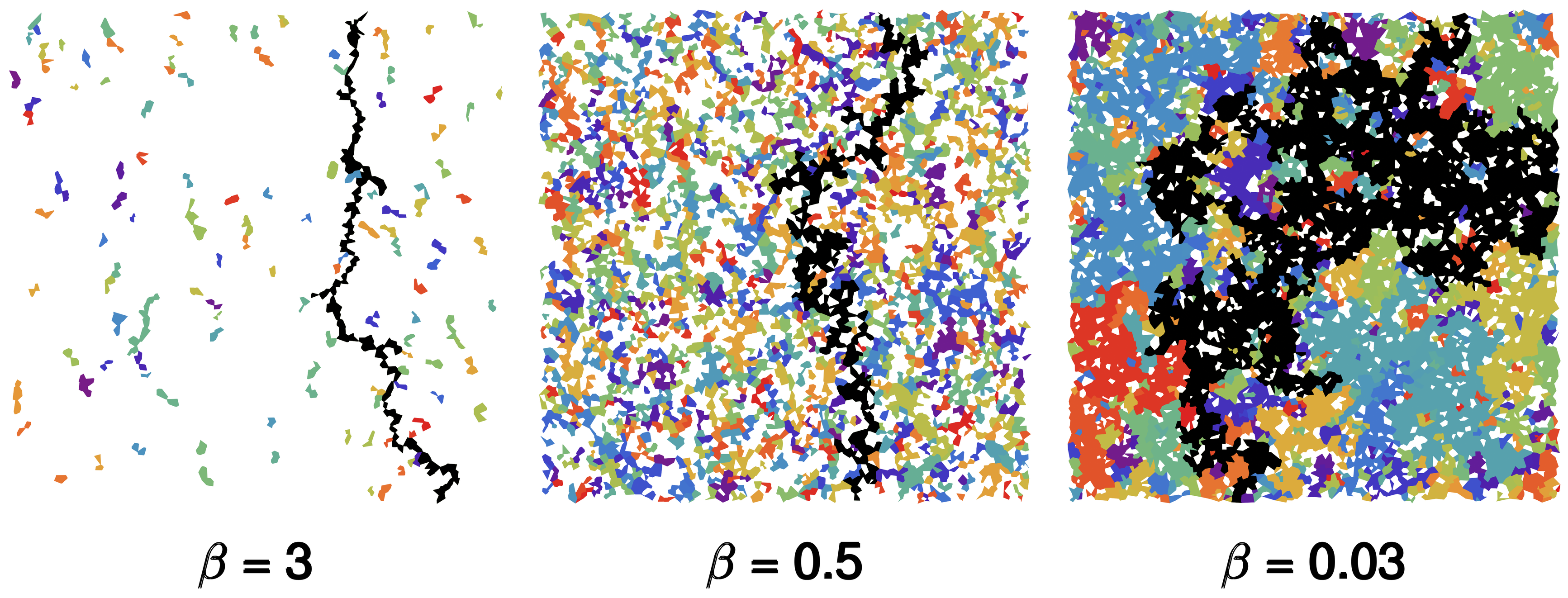

Figure 4: Fractured fuse networks at various β. Each colored region shows a contiguous cracked cluster. The black region shows the surface of the spanning crack.

Scaling in the Process Zone

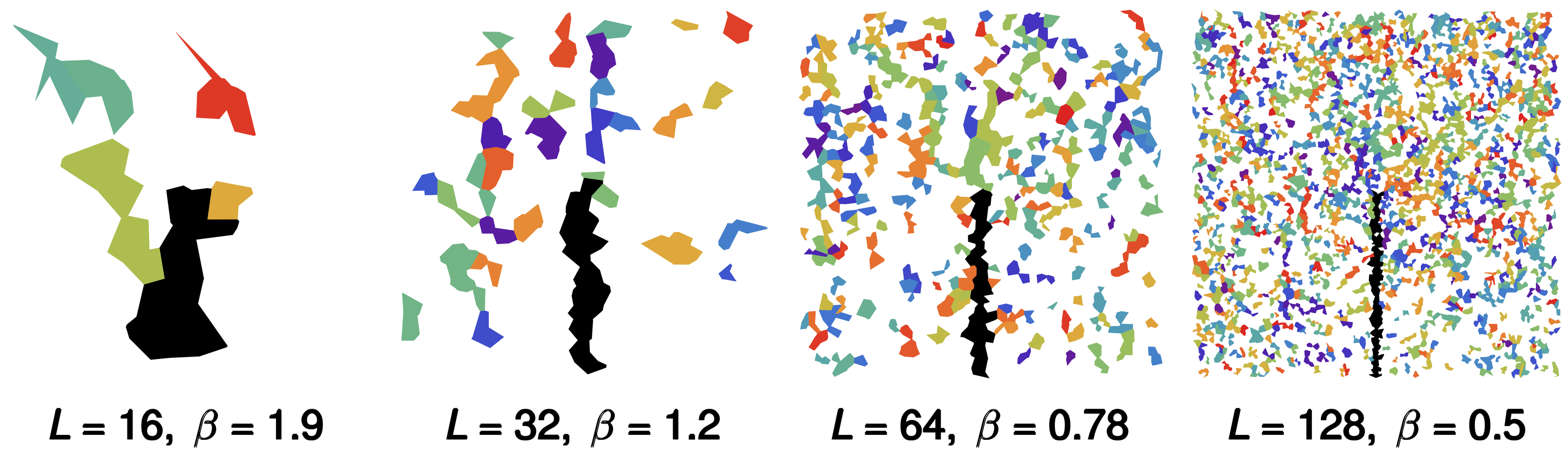

Figure 5: Notched fuse networks at critical stress. The disorder for each system is tuned so that $\beta L^{1/\nu_f}$ is constant, and the statistics of each should scale trivially.

We have made progress for developing a scaling theory of damage and stress in the process zone of quasibrittle cracks. We have essentially taken the scaling behavior of (1) as an ansatz for how the corresponding qualities of a critically semi-cracked network should scale. We have begun demonstrating the validity of this theory. Figure 5 shows critically cracked networks at constant $\beta L^{1/\nu_f}$, an invariant scaling combination under this theory. As can be seen in Figure 6, the disorder-averaged stress profiles caused by each collapses nicely.

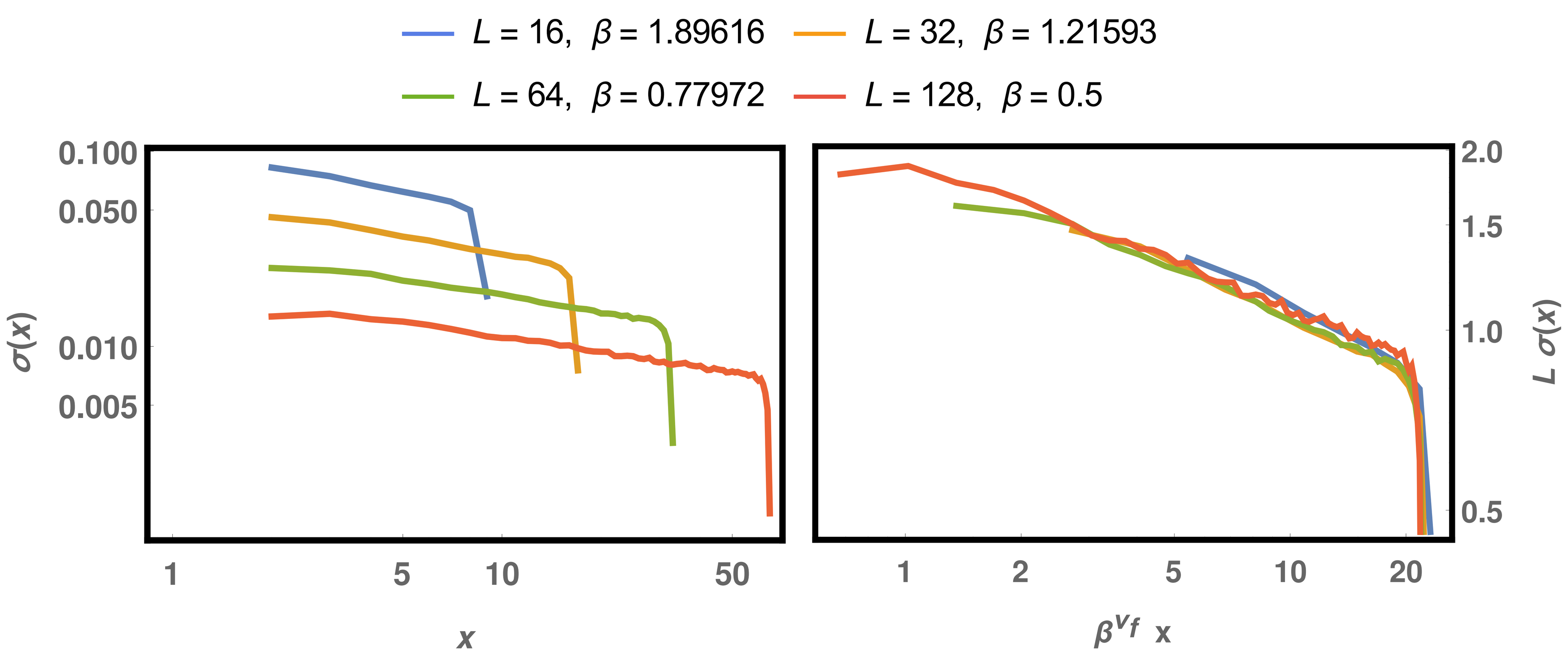

Figure 6: Disorder-averaged stress σ as a function of distance x from the tip of a critical crack. $\beta L^{1/\nu_f}$ is constant for each curve. **Left:** The unmodified stress. **Right:** The stress collapsed.

Next Steps

The voronoi networks we are able to generate allow us great flexibility for future multiscale computational modelling. Once we have hashed out our scaling theory more thoroughly, we plan on using it to probe much larger systems than previously feasible using networks whose fuse density becomes smaller in regions of less importance. This should allow us to see cleaner stress and damage scaling and cleanly stitch our discrete system to a continuum approximation.

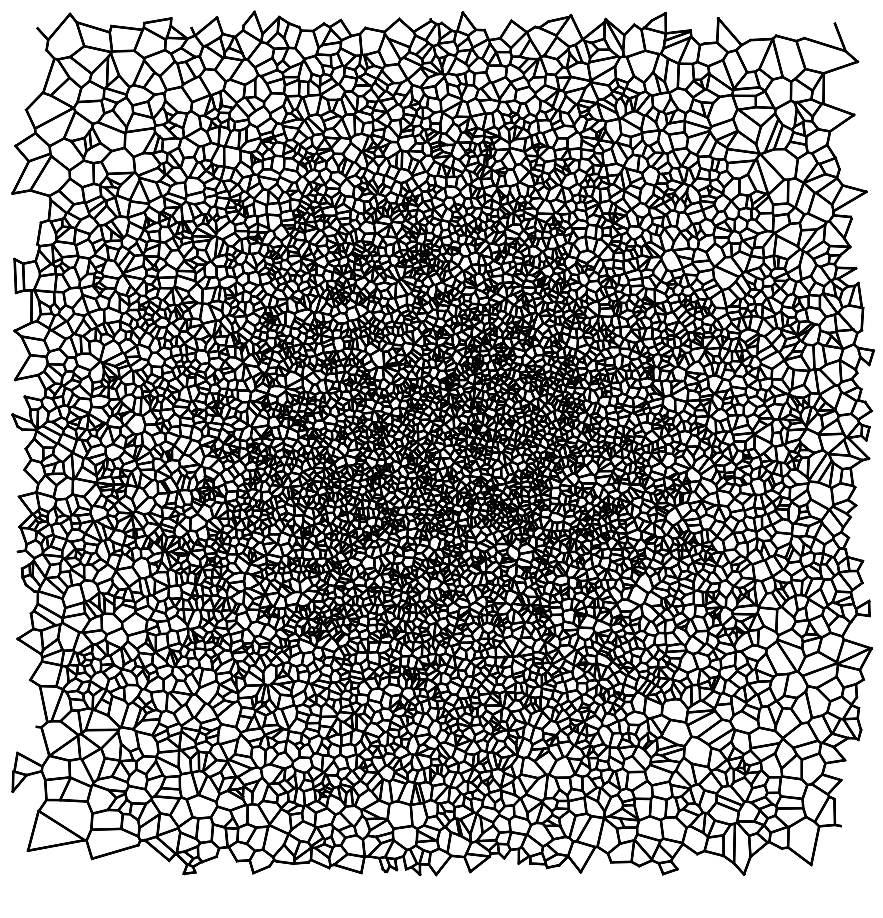

Figure 7: A hierarchical voronoi lattice.